THE QUANTITY THEORY AS TESTED BY KEMMERER.

THE ever-fluctuating production of the precious metals makes the discussion of price theory of ever-present practical interest and importance. The classical quantity theory of prices is now being considered in both scientific and popular publications. Almost invariably, this discussion of price theory revolves, first of all, about the question, what *is* the quantity theory and what are "the other things" that are supposed to "remain equal"? But whatever the conclusion may be as to the connection or lack of connection between the quantity of money and prices, or as to what is to be included in "the other things being equal," the next question that arises is, does the theory explain the facts?

Dr. E. W. Kemmerer, in his Money and Credit Instruments in their Relation to General Prices,[1] attempts, first, to find a mathematical statement of the quantity theory modified to suit present conditions, and, second, to test that statement by the available statistics. It is the purpose of this article to make a constructive analysis of the theory and statistics presented in this monograph. The analysis, therefore, divides itself into two parts, corresponding to Dr. Kemmerer's division.

I.

Has Dr. Kemmerer succeeded in his attempt to state the theory clearly and completely? In a review of the book Professor Laughlin says,[2] "We have here a defence of the quantity theory of money from the metaphysical school

[1] Published in 1907 by Henry Holt & Co. as Volume I. of the Cornell Studies in History and Political Science.

[2] Journal of Political Economy, November, 1907.

of economics, . . . a clear and admirable statement of the quantity theory." It is not quite certain what Professor Laughlin means by "metaphysical." If he means that the reasoning of the book is general, the epithet will be accepted readily. If he means that the conclusions have no scientific validity, the obvious reply is that the test is in facts which we shall consider later. As to the "clearness" of the statement, it is interesting to note the view of Mr. R. R. Hess, of the University of Wisconsin,[1] "Mr. E. W. Kemmerer has developed a formula which fairly illustrates the present intangible contentions of the supporters of the quantitative theory."

Dr. Kemmerer develops a price equation for a hypothetical society which is completely isolated and possesses wealth and an exchange system. There is no barter or credit; the money consists of ten thousand coins all alike; and the coins are issued solely on government account. Under the conditions, according to Kemmerer, "If we represent the quantity of money in circulation by M, the number of times it is turned over by R, the number of commodities exchanged by N, the number of times they are exchanged by E, and their price by P, it is evident that

$$MR = NEP; \text{ or (2) } P = \frac{MR''}{NE}$$ [2]

In deriving this equation, the further assumptions were made that the prices of all commodities were the same and that all were exchanged the same number of times. It may be remarked, in passing, that further explanation in regard to these assumptions is to be desired for the sake of clearness. But, even granting the assumptions, the equation cannot be correct if N represents the *number of commodities*. Does the *amount*, or number of units of each commodity changing hands, make no difference? If N should stand for the *number of units of commodity*, all units

[1] The Standard of Value and Prices, Journal of Political Economy, July, 1907, p. 399.

[2] Money and Prices, p. 13.

having the same average price and rate of turnover, the equation would be true.

Dr. Kemmerer later assumes that there are ten denominations of money and ten kinds of commodities in his hypothetical society, and "that each denomination of money and each variety of commodities has a rapidity of turnover peculiar to itself." [1] He uses the following symbols:—

$M_1, M_2, \ldots M_{10}$, to designate "the different denominations of money."

$R_1, R_2, \ldots R_{10}$, to designate "their respective rapidities of circulation."

$N_1, N_2, \ldots N_{10}$, to designate "the number of commodities of each variety to be exchanged."

$E_1, E_2, \ldots E_{10}$, to designate "their respective number of exchanges."

And combines them in the following price formula:—

$$P = \frac{M_1R_1 + M_2R_2 + \ldots + M_{10}R_{10}}{N_1E_1 + N_2E_2 + \ldots + N_{10}E_{10}}$$

This formula is supposed to be a mathematical statement connecting terms capable of exact quantitative expression. But the expression "the different denominations of money" is vague and has no quantitative meaning. Evidently, it is intended that M_1 should designate the amount, measured in terms of the money unit, of the coins of the first denomination in circulation; M_2, the amount of the coins of the second denomination, etc. For instance, if the first class consisted of 10,000 dimes and the unit were one dollar, M_1 would equal $1,000. Again, no definite meaning is given to "rapidity of circulation" until a statistical test of the equation is made on a later page. To be sure, he quotes Mill with approval in this connection as follows: "The phrase, rapidity of circulation, must not be understood to mean the number of purchases made by each piece of money in a given time. . . . The essential point is, not how often the money changes hands in a given time, but how

[1] Money and Prices, foot-note, p. 14.

often it changes hands in order to perform a given amount of traffic .... Some such expression as 'the efficiency of money,' though not unexceptionable, would do better; as it would point attention to the quantity of work done, without suggesting the idea of estimating it by time." That Mill does intend to bring in the time element is evident from the following, not quoted by Dr. Kemmerer: "We must compare the number of purchases made by the money in a given time, not with time itself, but with the goods sold in the same time."[1] The *exact* meaning attached to rapidity of circulation should have been given here by Dr. Kemmerer. Again, what can be meant by "the number of commodities of each variety"? If commodity is used to denote a class of goods, then "variety" is superfluous. It would seem that "commodity" is used in the sense of unit. We are told that P of the above equation stands for general prices, and that "the expression 'general prices,' as here used, represents a simple average of individual prices, and it is immaterial, so far as the demand for money is concerned, whether the individual prices, upon which the average is based, are all the same or all different."[2] It is impossible to agree or disagree with this statement until we are given more definite assurance of the meaning of "individual prices."[3]

Is it not possible to obtain a legitimate price equation between symbols having a definite mathematical meaning? Suppose that the following symbols stand for the concepts indicated:—

Let $M_1, M_2, \ldots M_{10}$, designate the average amounts, respectively, of the ten denominations in circulation, expressed in the money unit, during a unit of time.

[1] Mill, ii. p. 32. [2] Money and Prices, p. 14.

[3] Carver points out in The Distribution of Wealth (p. 4) that we should not pass to the explanation of the value of "things in general" before we explain the value of the unit of a particular commodity. Likewise the price equation should be built up from the price per unit of the commodity.

Let $R_1, R_2, \ldots R_{10}$, designate the average numbers of times, respectively, that coins of the first, second, third, etc., classes change hands during the unit of time.

$N_1, N_2, \ldots N_{10}$, designate the average numbers of units, respectively (bushels, yards, tons, etc.), of the first, second, third, etc., commodities in existence during the unit of time.

$E_1, E_2, \ldots E_{10}$, designate the average numbers of times, respectively, that units of the first, second, third, etc., commodities change hands during the unit of time.

$P_1, P_2, \ldots P_{10}$, designate the average prices, respectively, per unit of the first, second, third, etc., commodities during the unit of time.

Since it is assumed that all exchanges are made with the money, the following equation will be true:—

$$P_1N_1E_1 + P_2N_2E_2 + \ldots + P_{10}N_{10}E_{10} = M_1R_1 + M_2R_2 + \ldots + M_{10}R_{10}$$

On the left-hand side of the equation the products N_1E_1, N_2E_2, etc., represent what Irving Fisher calls the "flows" of the various commodities having prices, P_1, P_2, etc., per unit. These flows might well be represented by single symbols, $F_1, F_2, \ldots F_{10}$. As the various "flows" are here multiplied by the prices per unit of the various commodities, the total on the left-hand side of the equation is an expression in money units. And, since on the right-hand side of the equation amounts of the various denominations of money are multiplied by abstract numbers representing average rates, the total here is also an expression in money units. Since M_1, M_2, etc., are expressed in the same denomination, we can combine them, and let M designate

the sum $M_1 + M_2 + \ldots + M_{10}$. The total currency M will have some rate of turnover per unit which we will designate by R for the unit of time taken. For the summation on the right-hand side of the equation we can therefore substitute the product MR. If we let

$$P \text{ stand for } \frac{P_1N_1E_1 + P_2N_2E_2 + \ldots + P_{10}N_{10}E_{10}}{N_1E_1 + N_2E_2 + \ldots + N_{10}E_{10}}$$

where the flows N_1E_1, N_2E_2, $\ldots$ $N_{10}E_{10}$ may be looked at as being abstract numbers used as weights; if we let

$$NE \text{ stand for } N_1E_1 + N_2E_2 + \ldots + N_{10}E_{10},—$$

then the equation will be

$$PNE = MR.^{1}$$

In this equation P is the average price weighted by the flows of the various commodities. NE is the total flow of all commodities expressed as units of commodity.

Having obtained a statement of the quantity theory of prices for a hypothetical society, Dr. Kemmerer next proceeds to remove the restrictions and find how the statement would have to be changed by conditions as they exist in the United States at the present time. His treatment of the effect of "the other things" is admirable. The following is a summary:—

1. *Hoarding.* "Money therefore which is hoarded, and whose rate of turnover is zero, and likewise commodities which are not exchanged, have no numerical importance whatever in the price formula."[2]

2. *Barter.* "The exchange of goods by means of barter represents a demand for goods just as truly as does their exchange by means of money. Such exchanges affect the subjective valuations placed upon the goods by the various members of the community; . . . the changed subjective valuations above referred to would affect P through their influence upon E and R of the formula."[3]

[1] This form is derived by Kemmerer, but with different meanings given to the symbols.

[2] Kemmerer, p. 22. [3] Ibid., p. 25.

3. *Money in the United States at present.* Anything that acts as a medium of exchange of general acceptability is to be classed as money and included in M of the formula.[1]

4. *A Gold Monetary Régime.* "The possession by primary money of a bullion value essentially the same as its money value in no way exempts it from the general principle of price determination found to apply to other forms of money." [2]

5. *Credit.* Credit obligations may be classified as follows: "(1) where the obligation is in a form which is not negotiable, (2) where it is in the form of a negotiable instrument of postponed payment, (3) where it is in the form of a negotiable instrument payable on demand." [3] Credit obligations coming under heads (1) and (2), such as book credits and promissory notes, would have the same effect as barter. "The nature of the influence of checks upon prices is not essentially different from the influence of convertible government notes or bank notes." [4] Therefore, as Kemmerer states, the quantity theory equation will have to be changed by adding CR to MR and N_cE_c to NE and by substituting P_s for P where

C = volume of deposit currency exchanged for goods
R_c = average rate of turnover of such deposit currency
} = flow of deposit currency.

N_cE_c = the flow of goods exchanged for deposit currency.

P_s = the average price (weighted by the total flows) of all commodities sold for money and deposit currency.

The price equation then becomes

$$P_s = \frac{MR + CR_c}{NE + N_cE_c}$$ [5]

Does the new item, CR_c, denoting the flow of checks, vary independently of the money in circulation, M, or is it

[1] Kemmerer, p. 28. [2] Ibid., p. 60. [3] Ibid., p. 68.
[4] Ibid., p. 73.

[5] A similar form was derived by Irving Fisher in The Rôle of Capital in Economic Theory, Economic Journal, vii. p. 518.

a function of the money in circulation? Kemmerer supports the proposition that, "other things equal, the circulation of checks is a function of the monetary circulation."[1] "Assuming a given state of credit development, and a fixed amount of business, the proportion of deposit currency to bank reserves is a function of business confidence; and, business confidence remaining the same, an increase in the monetary circulation is accompanied by a proportionate increase in bank reserves and in the deposit currency which they support; a decrease in the monetary circulation has the opposite effect."[2] This conclusion is the result of an analysis of banking operations, which analysis is as convincing as it is clear.[3]

The quantity theory is a verbal statement of the price equation, and may be given as follows: *Conceive* the rapidity of circulation (R), the ratio of the flow of checks (CR_c) to monetary circulation (M), and the flow of commodities ($NE + N_cE_c$) to remain the same, general prices (P_s) will then vary in proportion to the monetary circulation. The truth of the price equation, however, is not dependent upon the assumption that certain terms be constants. All of the terms entering into the equation vary with time, although they may not vary independently of each other. With this understanding the price equation is inevitable. As to the *amount* of elasticity of the several variables and as to the correlation existing between pairs of variables entering the equation, there are very few data available, and consequently there is great difference of opinion among economists. The accumulation of statistical data bearing upon these points would seem to be a condition precedent to the further advance of price theory.

[1] Money and Prices, p. 78. [2] Ibid., p. 87.

[3] The subsequent statistical test given in this paper shows that facts bear out the theory.

II.

If we could obtain the average price level by weighting the prices of the individual commodities by the flows of such commodities for money and checks for a given year, if we could obtain the average amount of money in the country expressed in the money unit, if we could obtain the actual average rapidity of circulation per unit, if we could obtain the actual flow of checks and the actual flow of commodities for money and checks, and if these quantities should be substituted in the price equation, an identity should result. As a matter of fact, the flows of the various commodities cannot be obtained, the flow of checks can only roughly be estimated, and the flow of commodities for money and checks cannot be distinguished from the flow of commodities for commodities, or barter. It would, nevertheless, be both interesting and valuable to make a test of the price equation with such data as are available. Kemmerer now sets before himself the task of testing the equation by the statistics of actual business conditions.

Even among those who agree as to the statement of the quantity theory there is much difference of opinion as to the magnitude of the effect of the variations of the quantity of money upon prices. Professor F. M. Taylor says, "Experience shows quite conclusively that, in actual practice, the value of money exhibits little tendency to vary inversely as its quantity. Either the power of changes in quantity is very slight or it is commonly neutralized by opposing causes."[1] Further, he says, "It is always possible to conceive changes in the quantity of money so great that they would effect opposite changes in its value. Consequently, the quantity doctrine furnishes a convenient logical instrument for dealing with quite a number of money problems."[2] He doubts whether the increase of the output of gold following the Californian and Australian discoveries, or the recent increase following the South African

[1] Chapters on Money, p. 201. Published as a text-book in 1906.

[2] Ibid., p. 226.

and Alaskan discoveries, have caused a rise in prices.[1] In the same way Mr. A. S. Bolles expresses the opinion:[2] "The increase in the gold supply has no more effect in expanding business and raising prices than a thunder shower would have in raising the waters of the Atlantic."

On the other hand, Professor J. P. Norton holds not merely that the quantity theory is "a convenient logical instrument," but that the price changes at the present time are due primarily to the recent increase of the quantity of gold. Indeed, in his view, "So important is this subject that it would appear that Congress could well afford to appoint a commission to take testimony and to gather evidence in order to arrive at an adequate judgment as to the stability of the standard of value."[3]

The divergent views above quoted show the desirability of making a careful statistical test of the quantity theory. Kemmerer attempts to find the answer that facts give to the following questions:—

1. Do the bank reserves vary directly with the money supply?

2. Does the proportion of bank reserves to check circulation vary directly with the degree of business distrust existing in the country?

3. Is "a relative increase in the circulating media accompanied by a corresponding and proportionate increase in general prices and a relative decrease in the circulating media, by a corresponding and proportionate decrease in general prices," or, in the language of the formula, is

$$P_s = \frac{MR + CR_c}{NE + N_cE_c}$$[4]

borne out by the facts?

Dr. Kemmerer appreciates the complexity of the problem and the meagreness of the data and that any study of the relation between money and prices at the present

[1] Chapters on Money, p. 209.

[2] Journal of Political Economy, January, 1907.

[3] Yale Review, November, 1906.

[4] Money and Prices, p. 139.

time must be far from final. He says, "In a study of the type here undertaken, which attempts to measure numerically the relative movements of such factors in the problem as the monetary and check circulations, the amount of business done, business confidence, and general prices, *very rough approximations* are all that can be expected. That such is all the following study pretends to give cannot be insisted upon too strongly. The 'other things' that must be equal, in order that the proportionality of relation between the monetary circulation and general prices expressed by the quantity theory may be true, are altogether too numerous, too complex, and too closely interrelated to permit of anything like exact quantitative formulation." [1]

Dr. Kemmerer uses statistics of the United States for the period 1879–1904 to make his inductive tests. The statistics of total bank reserves he obtains from the Reports of the Comptroller of the Currency. For the amount of money in each year (M) he takes the average of the total money in circulation at the beginning and end of that fiscal year as given in the Statistical Abstracts. The check circulation for each year he obtains by multiplying the total bank clearings [2] in each year by $\frac{1}{35}$. This ratio is the ratio between the estimated [3] total check circulation for 1896 and the bank clearings for that year. The rapidity of circulation of 47 per year he derives by dividing the estimated total money transactions in 1896 [4] by the money circulation of that year. The figures for the growth of business he finds by taking the simple average of index numbers of fifteen different series of statistics taken as

[1] Money and Prices, p. 90.

[2] The figures for yearly clearings of the United States were obtained from The Commercial and Financial Chronicle and Monthly Summary of Commerce and Finance of the United States.

[3] This estimate was based on reports made by a large number of banks of the country of the proportion of money and checks in deposits made by their customers, retail, wholesale, and others, on the settling day nearest July 1, 1896. Report of Secretary of Treasury, 1896, p. 456.

[4] He computes the total money transactions by taking one-third of the estimated check transactions.

representing the industrial activity of the year considered.[1] "The index numbers of business distrust are the simple averages of the corresponding indices for the proportion of concerns failing, and the average liabilities of concerns failing." [2] "The general index figures of prices and wages were computed by combining in a weighted[3] average the index figures for the prices of railroad securities (Commons), the index figures for the prices of wholesale commodities (Commons), and the index figures for wages (Department of Labor tables for twenty-five occupations)." [4]

All of the questions to be tested by the statistics collected are questions of correlation. Dr. Kemmerer makes the tests graphically by comparing the fluctuations of the two curves based upon the pair of series of statistics being considered. Such a method is well enough as a preliminary, but it tells nothing of the *extent* of the correlation between the series of figures being considered. The charts presented by Dr. Kemmerer do not answer the questions that he asks. A numerical measure of the correlation must be found if we wish to determine the *extent* to which the fluctuations of one series of statistics synchronize with the fluctuations of another series.

[1] Index numbers (with the average for the years 1883–84–85 as the base, of the following series of statistics are simply averaged to give index numbers of the growth of business:—

1. Population.
2. Tonnage entered and cleared.
3. Exports and imports of merchandise.
4. Revenues of Post-office Department.
5. Gross earnings from operation of railroads of the United States.
6. Freight carred by the railroads of the United States.
7. Receipts of the Western Union Telegraph Company.
8. Consumption of pig iron.
9. Bituminous coal retained for consumption.
10. Consumption of wheat.
11. Consumption of corn.
12. Consumption of domestic and foreign cotton.
13. Consumption of domestic and foreign wool.
14. Consumption of wines and liquors.
15. Market value of reported sales of the New York Stock Exchange.

[2] Money and Prices, p. 125.

[3] The weights are: 3 per cent. for stock; 8 per cent. for wages; 89 per cent. for prices.

[4] Ibid., p. 136.

The coefficient of correlation "serves as a measure of any statement involving two qualifying adjectives, which can be measured numerically, such as 'tall men have tall sons,' 'wet springs bring dry summers,' 'short hours go with high wages.'"[1] In the statistical test of the quantity theory we wish to determine whether high prices go with a high relative circulation. The method of measuring correlation which will be used here to test Dr. Kemmerer's statistics is applicable to cases in which the distribution of the items is either symmetrical or skew.[2] In order to measure the correlation, it is necessary to compute the arithmetic averages of the two series of statistics to be compared, the standard deviations, and the probable errors.[3] When there is perfect direct or positive correlation, the coefficient of correlation will equal $+1$; when there is perfect inverse or negative correlation (an increase in an item of one series coming with a decrease in the corresponding item of the other series), the coefficient will equal -1; when there is absolutely no correlation, the coefficient will equal 0.

The probable error gives a measure of the unreliability of any determination. Its meaning can be illustrated as follows. Suppose that we wish to determine the average height of the 22,000,000 men in the United States. We measure the heights of (say) 100,000 men chosen at random

[1] Bowley, Elements of Statistics, p. 320.

[2] Davenport, Statistical Methods, p. 42.

[3] The formulæ used are as follows:—

Arithmetic mean $(m_x) = \frac{\Sigma x}{n} \pm$ a probable error of $0.6745 \frac{\sigma_x}{\sqrt{n}}$

Standard deviation $(\sigma_x) = \sqrt{\frac{\Sigma v_x^2}{n}} \pm$ probable error of $0.6745 \frac{\sigma_x}{\sqrt{2n}}$

Coefficient of correlation $(r_{xy}) = \frac{\Sigma(x - m_x)(y - m_y)}{n\sigma_x \sigma_y} \pm$ a probable error of $\frac{0.6745 (1 - r_{xy}^2)}{\sqrt{n}}$

Where x = numerical measure of one item

n = number of items.

V_x = algebraic difference of an item (x) from the arithmetic mean $(m_x) = x - m_x$.

from the whole group, and find the arithmetic average and the probable error of the arithmetic average, using the formulæ given in the foot-note. If the arithmetic average came out 5 ft. 7 in. and the probable error 1 in., it would mean that chances are even that the true average lies between 5 ft. 6 in. and 5 ft. 8 in.

The following table gives the correlation coefficients for various pairs of series of statistics for which Kemmerer has attempted to show the correlation by means of charts:—

TABLE SHOWING THE ARITHMETIC MEANS, STANDARD DEVIATIONS, COEFFICIENTS OF CORRELATION, AND PROBABLE ERRORS OF CERTAIN FINANCIAL STATISTICS.

(*Statistics from Kemmerer's Money and Prices, p.* 141.)

Statistics of	Period Covered.	Arithmetic Mean (m) ±Probable Error.	Standard Deviation (σ) ±Probable Error.	Coefficient of Correlation (r) ±Probable Error.
Money in circulation, inclusive of bank reserves	1879–1904	123.9±4.4	32.9±3.1	(Immediate)
Bank reserves	" "	150.9±7.1	53.4±5.0	+0.979±0.006
Business distrust	" "	85.8±1.9	14.3±1.3	(Immediate)
Ratio of bank reserves to check circulation	" "	89.6±2.7	20.3±1.9	+0.53±0.095
Business distrust	1879–1903	86.0±2.0	14.6±1.4	(Anticipatory)
Ratio of bank reserves to check circulation	1880–1904	89.9±2.8	20.7±2.0	+0.723±0.064
Relative circulation	1879–1901	107.0±1.7	12.1±1.2	(Immediate)
General prices	" "	91.3±1.5	10.5±1.1	+0.23±0.13
Per capita circulation (Statistical Abstract)	" "	23.1±0.32	2.24±0.23	(Immediate)
General prices (Kemmerer)	" "	91.3±1.5	10.5±1.1	−0.18±0.14

In the case of the correlation of bank reserves and money in circulation, inclusive of bank reserves, Dr. Kemmerer says, "There can be no question but that when due allowance is made for fluctuations in business confidence, the evidence . . . strongly supports the contention that there exists a close relationship between the amount of money in circulation and the amount of the country's bank reserves."[1] The coefficient of correlation comes out to be 0.979 ± a probable error of 0.006, which shows a most

[1] Money and Prices, p. 143.

remarkable correlation.[1] "When r is not greater than its probable error we have no evidence that there is any correlation, for the observed phenomena might easily arise from totally unconnected causes; but, when r is greater than, say, six times its probable error, we may be practically certain that the phenomena are not independent of each other, for the chance that the observed results would be obtained from unconnected causes is practically zero."[2]

The correlation coefficient between the index numbers of business distrust and the ratio of bank reserves to check circulation for the same years is 0.53 ± 0.095. When the index numbers of business distrust for one year are correlated with the ratio of bank reserves to check circulation the following year, the coefficient is 0.723 ± 0.064. As Dr. Kemmerer has suggested (but not verified), there is a closer correlation "when proper allowance is made for the time required for alterations in business confidence to exert their influence on bank reserves."[3] A very decided correlation is shown by the coefficient.

The final test of the quantity theory is the correlation between the figures for the right and for the left hand sides of the equation $P_s = \frac{MR + CR_c}{NE + N_cE_c}$. Upon examination of the curves plotted from the two series of statistics representing general prices and relative circulation (the left and right hand sides, respectively, of the price equation) Dr. Kemmerer says, "The general movement of the two curves

[1] For purposes of comparison the following correlation coefficients are given:—

1. Certain correlation coefficients for man:—
   - Right and left femur 0.96
     (the highest in list of correlation of human measurements.)
   - Weight and stature of Cambridge (England) female students, 0.721 ± 0.026
   - Weight and stature of Cambridge (England) male students, 0.486 ± 0.016
   - Forearm and stature 0.37
   - Age at death of consorts 0.22
   - Longevity of father and son 0.12

   Davenport, Statistical Methods, p. 75 *et seq.*
2. Certain correlation coefficients of New York Money Market:—
   - Ratio of reserves to deposits and discount rates (780 weeks), 0.52 ± 0.01
   - Reserve and loan periods (immediate) . . . . . 0.49 ± 0.07
   - Reserve and loan periods (after three weeks) . . . . . 0.96 ± 0.01

   Norton, New York Money Market, p. 96.

[2] Bowley, Elements of Statistics, p. 320.

[3] Money and Prices, p. 146.

taken as a whole is the same, while the individual variations from year to year exhibit a striking similarity."[1] However, the correlation coefficient for these series of statistics is only 0.23 and has a probable error of 0.13. The probable error is more than half the correlation coefficient, showing that the chances are even that the true coefficient lies between $+0.10$ and $+0.36$. The chances are about 4.5 : 1 that the true correlation coefficient lies between zero and $+0.50$. A small positive correlation is thus indicated. The contention of the quantity theorists is that, if we could obtain the exact measures, for a series of years, of all the items appearing in the price equation, the coefficient of correlation between the right and left hand sides would be $+1$. The smallness of the correlation indicated may have resulted either because the theory is in error or because the statistics are not adequate to test the theory. Whatever may be the fact, the statistics presented by Kemmerer do not demonstrate that general prices move in sympathy with relative circulation. However, he was under the necessity of assuming that the rapidity of circulation remained constant during the period and of estimating the volume of check transactions upon a basis of one day's business. He was also obliged to use index numbers based upon a miscellaneous lot of industrial statistics to represent the flow of commodities. In this substitution a correlation was assumed (altho necessarily) which, if not perfect, would seriously interfere with the correlation that he is attempting to measure. In this connection it should be noted that Kemmerer has obtained a correlation coefficient of 0.23 ± 13, whereas the coefficient of correlation of the per capita circulation figures[2] and general prices[3] is negative, being -0.18 ± 0.14.

In conclusion, it may be said that Kemmerer's investigations have shown that the questions to be answered must be answered by induction through the collection of adequate statistics. It is to be hoped that studies of this nature will be undertaken.

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[1] Money and Prices, p. 147.

[2] Statistical Abstract.

[3] Kemmerer's index numbers.